

63. (a) The force acting on the satellite has magnitude GMm/r^2 , where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r , where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite: $r = 6.37 \times 10^6 + 640 \times 10^3 = 7.01 \times 10^6$ m. Thus,

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{7.01 \times 10^6 \text{ m}}} = 7.54 \times 10^3 \text{ m/s} .$$

- (b) The period is $T = 2\pi r/v = 2\pi(7.01 \times 10^6 \text{ m})/(7.54 \times 10^3 \text{ m/s}) = 5.84 \times 10^3$ s. This is 97 min.
 (c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4 \times 10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.

The initial energy is

$$E_0 = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(7.01 \times 10^6 \text{ m})} = -6.26 \times 10^9 \text{ J} ,$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26 \times 10^9 \text{ J} - (1500 \text{ orbit})(1.4 \times 10^5 \text{ J/orbit}) = -6.47 \times 10^9 \text{ J} ,$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(-6.47 \times 10^9 \text{ J})} = 6.78 \times 10^6 \text{ m} .$$

The altitude is $h = r - R = 6.78 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m} = 4.1 \times 10^5$ m. Here R is the radius of Earth. This torque is internal to the satellite-Earth system, so the angular momentum of that system is conserved.

- (d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.78 \times 10^6 \text{ m}}} = 7.67 \times 10^3 \text{ m/s} .$$

- (e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78 \times 10^6 \text{ m})}{7.67 \times 10^3 \text{ m/s}} = 5.6 \times 10^3 \text{ s} .$$

This is equivalent to 93 min.

- (f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7$ m, and $\Delta E = -1.4 \times 10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N} .$$

- (g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.
 (h) The satellite-Earth system is essentially isolated, so its momentum is very nearly conserved.